notches, etc.) this approximate method of determining K_p and K_ϵ may become increasingly more inaccurate and its usage should be restricted to relatively small and moderately small strain ranges.

Discussion

Recently Ishikawa[5] has examined the problem of plastic stress concentration around a hole in an infinite sheet under equal biaxial tension. His anlaysis makes use of the deformation theory of plasticity and the analytical results given include plastic stress concentration factors and stress fields.

The present method is capable of predicting only plastic stress and strain concentration factors at the discontinuity, but is not restricted to a particular value of K_{el}. The method therefore is only applicable for studying the local concentrations at the discontinuity.

For a value of $K_{el}=2$ we have shown in Fig. 2 a comparison between results of the proposed simigraphical method and results obtained by analytical methods[5,6] for K_p. The agreement between K_p values at various levels of applied stress is quite good. The maximum difference between our results and those of Ref. 5 is of the order of 10%. The agreement with the values of Ref. 6 is somewhat better.

Conclusions

1) It is suggested that the approach proposed in this note is quite general, since the type of loading and geometry are contained in the value of K_{el}. 2) Experimental verification would be necessary to establish the limitations of the proposed method.

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A Practical Aspect of Bass' Algorithm for Stabilizing Linear-Time-Invariant Systems

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IN the design of modern flight control systems whose dynamic behavior is described by the vector differential equation

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{1}$$

it is often required to compute a full-state feedback control law of the form

$$u(t)=Kx(t) \tag{2}$$

such that $(A+BK)$ is a stability matrix. A particular case of such a requirement is the solution of the steady-state Riccati equation by Newton's method[1] for aircraft with relaxed static stability.

Kleinman[2] and Bass[3] have proposed algorithms to generate stabilizing control laws. Bass' algorithm, however, has the advantage of simplicity and computational ease. The Bass algorithm calls for a solution of a Lyapunov type matrix equation. This is stated in the following theorem.[3]

Theorem—Bass Algorithm:

Let A, B be a controllable pair, then

$$K=-B'Z^{-1} \tag{3}$$

stabilizes the system (1) where $Z=Z'>0$ satisfies

$$-(A+\beta I_n)Z+Z[-(A+\beta I_n)]'=-2BB' \tag{4}$$

with $\beta>\|A\|$, where $\|\cdot\|$ is any vector-induced matrix norm,[4] and $(\cdot)'$ indicates the transpose.

The purpose of this Engineering Note is to provide a simple and computationally tractable value for the scalar β directly in terms of the elements of the A matrix. Obviously, if $\beta>\|A\|$, then the matrix $-(A+\beta I_n)$ is a stability matrix; therefore, any value of β which makes $-(A+\beta I_n)$ a stability matrix will satisfy the requirement for the Bass algorithm.

Let P be the modal matrix of A. Then A is given by

$$A=P\Lambda P^{-1} \tag{5}$$

where

$$\Lambda=\operatorname{diag}(\lambda_1,\lambda_2,\ldots\lambda_n) \tag{6}$$

With no loss of generality, let λ_1 be the eigenvalue of A with the largest absolute value real part, namely,

$$|Re(\lambda_1)|\geq|Re(\lambda_i)|\,\forall i \qquad i\neq 1 \tag{7}$$

Therefore, if $\beta>|Re(\lambda_1)|$, the matrix $-(A+\beta I_n)$ is a stability matrix. Applying the Gersgorin Theorem,[4] we readily obtain

$$|Re(\lambda_i)|\leq\max\left|a_{ii}+\left[\operatorname{sgn}(a_{ii})\right]\left(\sum_{j,j\neq i}|a_{ij}|\right)\right| \tag{8}$$

a_{ij} is the (i,j) element of the matrix A. Consequently, β in Eq. (4) can now be specified as

$$\beta=\eta\max\left|a_{ii}+\left[\operatorname{sgn}(a_{ii})\right]\left(\sum_{j,j\neq i}|a_{ij}|\right)\right| \tag{9}$$

where $\eta>1$.

A simple method has thus been established to determine the parameter β in the Bass algorithm. Equation (9) is amenable to simple machine calculations.

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